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# Games Thinkers Play 

Presented by:<br>Mary Behr Altieri<br>Putnam/Northern Westchester BOCES<br>Yorktown Heights, New York

maltieri1212@gmail.com

## The Role of Games in the Mathematics Classroom

With a packed curriculum and a wide range of ability in most math classrooms, who has time for games? Early in my teaching career I became convinced that math games could become a significant tool and I made time for them. I learned from my success with children playing games, and learned even more from my mistakes. I share that experience with other teachers so they don't have to waste time making the same mistakes I did. They can be creative and make their own!

## Games of Chance

Winning some childhood games is completely dependent on chance. The card game called "War" is a good example. Once you have the knowledge and skill to play the game (knowing the order of values of the cards) you can only "play the cards you're dealt" and you have no control over the outcome. You can just as easily win as lose. You don't learn to get "good" at games like this. You can't improve your chances of winning by developing any knowledge, skill or strategy. Chance games require low knowledge levels and practice is therefore also low level.

## Content Knowledge Games

Winning other games is almost entirely focused on content knowledge with a speed sometimes added as a factor. "Jeopardy" is a good example of this type of game. The person who can tap quickly into the knowledge required will win the game. S/he can improve the chance of winning by increasing knowledge and speed. Content knowledge games are won by the more able student. The less able student is usually not a fan of this type of game. The able student can become bored by it unless s/he is matched well with an opponent, which is not so easily accomplished.

## Strategy Games

Winning at chess is a different matter. It is strategy, the skill of playing the game itself, that determines the winner. Developing strategies, practice of others' winning strategies, concentration and thinking ahead are some important skills for these types of games. Strategy games help develop a number of good skills for a person to have. However, these skills may not be as content oriented as a time constrained curriculum requires.

Good games for mathematics are games that are not purely any one of the types described. They are games that combine the best qualities of each, and then add in some other factors to make them teacher- and classroom-friendly.

## Characteristics of Really Good Math Games

- Kids like to play them.
- They are games that have mathematical content at their core.
- Playing the game encourages and/or provides skill practice and concept development.
- The games involve a combination of chance, content skills/concepts, and strategic possibilities.
- The games vary in complexity for different players depending upon their level of skill and understanding of content and strategy. (Think about how chess is different for players with different skill levels.)
- After initial teaching, the games can be played relatively independently with readily available and inexpensive materials.
- They can be "morphed" to meet the needs of students, thus providing an opportunity for differentiating instruction.

Generally a really good math game is one that provides fun and learning at the same time. A player can improve his/her chance of winning by content knowledge and by strategy. However the chance factor always plays a role and sometimes allows a less able player to win. Playing the game provides practice and/or allows opportunities to see math relationships in a different way. Because it is available to students, and because they like to play it, the game becomes part of the students' math resource materials, helping them to become better math students.

## More than Half

Materials: One game sheet, one pair of dice, and two different colored crayons for each pair of players.
Object: This game is best played as a two-person game. The object of the game is to own more circles than one's opponent. A player owns a circle when more than half of it has been colored with her color.

## Directions for the Basic Game:

- Players roll one die to see who becomes "Player One".
- Player One rolls the pair of dice. The lower number represents the numerator, and the larger number represents the denominator. For example, if a 3 and a 5 are rolled, the number rolled is the fraction 3/5. Player One says "three fifths" and colors in three of the pieces that represent $1 / 5$.
- Player Two then rolls the dice and colors in the pieces that represent her roll.
- The players take turns rolling the dice and naming and coloring each fraction that is rolled.
- Each player must use all of her roll. If she cannot use her whole roll, then she loses a turn.
- On each turn a player may color all of her roll on one circle, or split it among two or more circles.
- When a player has more than half of any circle in her color, then she puts her initials next to that circle. (This initialing makes counting circles at the end easier, but does not mean that blank parts of the circle can't be used when necessary to complete a person's roll. This, of course, does not affect ownership.)
- If a circle is colored equally by two players, each having half, then neither player may claim the circle. The circle is neutralized.
- Play continues until all circles have been claimed or neutralized, or when it becomes obvious that one must player must win.


Adapted by M. Altieri from Open Court's Real Math Fraction Game

More Than Half--Harder Version


Adapted by M. Altieri from Open Court's Real Math Fraction Game

## From Here to There

These games are rather quick ones that provides practice in counting by tens and ones from any number. There are some strategies involved that are rather sophisticated. However, knowing the strategies is not required to play the game. Strategies just enhance one's chance of winning.

They are all two player games.
In the first version, each player starts with ten dimes and ten pennies. Players take turns contributing one, two or three coins to the "pot". On placing the coins in the pot, the player calls out the cumulative total after each coin s/he places. The person who puts in the coin to make the total value of the pot exactly one dollar is the winner.

## Variations:

- Play the same game with dimes and pennies, but have students put them on a hundreds chart instead of in a "pot." This helps those students who still struggle with counting by tens and ones, or have difficulty remembering or hearing the other player's numbers.
- Play the game on the hundreds chart without the dimes and pennies --no it is just getting to one hundred by tens and ones--but just keeping track tens and ones by marking the increasing "pot" on the chart.
- Play the game with no coins and no hundreds chart. Students just take turns saying and writing each number in two different colors (one for each player) to keep track. A sample of a completed game of this variation follows.

$$
10,11,21,22,23,33,34,44,54,64,65,75,85,8696,97,98,99,100
$$

- Play the game the same way as above, but with the two players just saying the numbers and not recording them.
- Play the game the same way as either of the previous two, but start with a number other than one, and end with a number other than 100.
- Play the game the same way as any of the previous three, but using ones, tens and hundreds OR ones, tenths and hundredths.
- Play the game the same way as any of the previous, but use up to 2,4 or 5 coins or numbers at a time.

There are more variations to this game than can fit on this paper. Your students may like creating variations as well. Have fun experimenting!
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## HUNDRED CHART

| $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
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| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |


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| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

## ORDER



## GAME



| FINISH |
| :---: | :---: |

## Order Game Fraction/Percent Version

This is a game in which students can use their knowledge of fraction/decimal/percent equivalents, and use of strategy to complete the game board.

Materials: Three 0-5 number cubes and the game board. (The purpose of the game board is to determine the number spaces to be filled before a game is complete. The game board can be changed to more or fewer spaces to make it more or less difficult.)

The object of the game is to be the first to have all the spaces filled with each percent greater than the value below it. The topmost space must be filled with a percent that is no greater than 100.

## Play:

Player One: Rolls the three number cubes. S/he then uses two of the number cubes to make a percent or a fraction (which s/he then changes to a percent). Player One places that percent in any space on her/his column.

Player Two: Also rolls three cubes and chooses two of the rolled numbers to make a percent, or a fraction to change to a percent and places it in any space in her/his column.

Play continues until one person has all the spaces filled.

If there is no possible value to be made that will fit in an empty space, then the player loses her/ his turn.
(For the first several times the game is played, a teacher may choose to have the fraction/ percent equivalents posted in the room. After a few times, most students no longer need that help. Posting is preferred to having them in the student's immediate view so that the student has the opportunity to think before referring to the posting.)
Pennies/Dimes/Dollar GAME BOARD



TENS

## Cooperation

Materials: Index cards for each student. Each index card shows a number from 1 to 9 . The cards are then punched and "strung" so students can hang them around their necks.

Object: The object of the game is to have students join with other students to form expressions equal to a given target number within a given time.

Scoring: At the end of each round, each student in a given expression scores the number of points equal to the number of people in the expression.

Sample Game: Leader passes out a number card to each student. The number card is hung around the neck so that each student can see each other student's number.

## Round One:

The leader announces a time limit; for example, two minutes, and a target number; for example, 12.
Students then move around looking for other students with whom they can join to complete an expression (using addition, subtraction, multiplication, and/or division) whose value is 12.

Below are examples of the numbers in an expression and how each team created the expression.

| 2 | 3 | 4 | 2 |  | because $2 \times 3+4+2=12$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | 3 | 1 | 1 | 4 | because $(9 / 3) \times 1 \times 1 \times 4=12$ |
| 6 | 5 | 3 | 9 | 2 | 6 |

The number of points for each student in the first expression is 4 because there are four students representing the expression. Therefore, the number of points for each student in the second expression is 5 , and the score in the third expression is 6 .

Each student keeps track of his/her own points. The leader begins round two. Game continues for the time allotted. Students with the most points are the winners. There are usually lots of winners!

Suggestions:

- When played early in the year, it becomes clear who the leaders are. It is probably a good idea to give the most valuable number (1) to the least popular child. Then all of the "expressions" will want him/her!
- The game changes as the students progress in their understanding of the properties of numbers, so it is best played at several different times during the year.
- If students are getting too hung up on addition and subtraction only, try offering bonus points for each multiplication and/or division in an expression.
- It is important to have each student participate in the saying of how his/her number belongs in the expression, rather than have one student do it all.
- Some students use their arms to create parenthesis around certain sections of the expressions. Other students stand on tables (use your judgment) with other students under the tables to express division!
- It might be interesting to make one or two of the index cards "wild cards."

I do not know the origin of this game. I don't even remember who taught it to me. But I thank them wherever they are!

# Roll to Fifteen 

## Materials:

20-5 number cubes
25-10 number cubes

Number line to 20 (optional)

This is a game for two players.

Player one rolls a cube, looks a the result and rolls another to be added to it. No cube may be rolled more than once.

Because the goal of the game is to get as close to 15 as possible, the player stops rolling when s/he gets as close as s/he thinks is possible.

Then player two rolls the cubes, one at a time, trying to get closer to 15 than player one.

This is all mental math. The number line is for those students who have difficulty determining which of two numbers is closer to 15, especially when one is over and one is under.

Variation: Twenty to Five
Same game but going backwards from 20 and trying to get as close to 5 as possible.

[^0]
## Roll a Problem Game

This game is played on a simple grid, as shown below, with two number cubes. One of the cubes (known as the low cube) is numbered from 0-5, the other (the high cube) is numbered from 5-10. In this game if a 10 is rolled, it is "wild." Each player may us it as any digit from 0 through 9.


Each player has his/her own grid. A number cube is rolled. The sequence of rolls for this game is: High, Low, High, Low, High, Low. All players use the number that is rolled to occupy one of the spaces on the grid. Once a number is placed in a space, it may not be moved. All players must place their numbers after each roll, before the next cube is rolled. When six numbers have been rolled, the grid will be filled in and each player adds the two three-digit numbers. The player with the highest sum is the winner.

There are many variations of this game. The number of digits can change, the cube sequence can change. The object of the game can change, AND, the operation can change as well.

An interesting variation that is a favorite among my students is called, Don't Go Over a Thousand. The game is played as above except that the object of the game is to have the highest sum which is no greater than one thousand.

Another interesting variation is to roll an extra cube, e.g., 7 cubes for 6 spaces. In that game, one of the digits rolled can be "discarded" before it is placed if it is judged to be a poor choice for the player.

Make sure you take some time, not necessarily during each game, to discuss the math. Some example questions for discussion of an addition game are below:

[^1]
## DIE PLOT

Die Plot is a game which helps reinforce skill in plotting ordered pairs in the first quadrant of a coordinate planar graph. In Game 1, only luck is necessary to win, but it is a good introduction to Game 2. Game 2 continues reinforcement and practice in plotting points, but also involves strategy. Although luck is still a factor, offensive and defensive strategic play help increase one's chance of winning.

## GAME ONE

Materials: 1 die plot grid for each pair of students
2 die of different colors

## Instructions:

Students are reminded that the two axes are the horizontal $x$-axis and the vertical
$y$-axis. An ordered pair is always in the form $(x, y)$; that is, the movement is along the $x$ - axis first, then the $y$-axis. The ordered pair $(2,3)$ is over 2 , then up 3. It is marked on the grid below.

In advance of play, the dice are given " $x$ " or " $y$ " designations according to color. For example, if playing with red and green, the red die may be designated as " $x$ " and the green as " $y$ ".

The game is played by each player, in turn, tossing the dice and "capturing" the point designated by the ordered pair. (Players can use x and o or different colors to distinguish their respective captured points."

Once a point is captured, it cannot be taken by another player. If a player rolls an ordered pair matching a point already captured, that player loses his/her turn. Play continues until one player has captured three adjacent points in a horizontal, vertical or diagonal row.


## GAME TWO

Materials: 1 die plot grid for each pair of students
2 PAIR OF DICE, each pair a different color

## Instructions:

The game is played similarly to Game One, the difference being that each player tosses two pair of dice. Again the dice have been designated $x$ or $y$ by color. The toss then yields two numbers of one color and two of another giving four possible ordered pairs.

Each player in turn tosses the dice and chooses which ordered pair would be most advantageous. If none of the ordered pairs appears to be particularly advantageous, the player may choose the point which most interferes with his/her opponent's play.

The object of the game remains the same. (Some choose to lengthen the game to four points in a row.)
I have always tried to give credit to whomever taught me a game. But I can't find the author of this one. I came upon it in an activity book from Creative Publications in the early 1970s. I think the name of the book was Eureka. So thanks to the creator!


## OH NO! 99!

This game provides practice for addition of whole numbers and strategic thinking.

## Materials:

A deck of playing cards (no jokers) for each pair of players.

## Card Values and Operations:

| Aces | +1 |
| :--- | :--- |
| Jacks | -10 |
| Queens | wild (can represent any other card in the deck) |
| Kings: | 0 |
| All others: | + face value |

## Directions for Play:

- One player shuffles the cards and deals four cards to each player. The cards not dealt remain in a stack, face down.
- Players take turn playing one card at a time, adding or subtracting the value of their card to or from the jointly accumulating score.
- Each time a player plays a card, he or she must replace it with the top card on the face-down stack.
- Play continues until one player forces his or her partner to go over the score of 99.

The way the game was originally intended players cannot see each other's cards. However, if players can see each other's cards the strategies require an interesting change.

## Management Tips and Techniques

. Build a repertoire of games.
. Provide borders for playing area.
. Create consequences for careless play.
Color code decks for return to original decks.
. You and me against the world.
. Save some for later--rules that is!
. Review with previously learned games.
. You learn from their play.
. Be prepared! Play it first.


[^0]:    This game, without a number line is originally from Real Math from Open Court.

[^1]:    Does everyone with the same sum have the same addends? Can you be sure you are a winner before you add? Do you know you have lost before you add? If you knew the numbers in advance, could you have created a winning sum? With 6 different digits how many different sums are there? What is the greatest sum possible in the greatest sum game? It is possible with rolling the cubes in the sequence above to have a sum of exactly 1,000 ?

